Lab 5:
Monte-Carlo Integration

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Abstract

The Monte-Carlo method of integration allows us to calculate amount of space something occupies knowing nothing more than its boundaries. In the simple case of estimating the area of a cicle/sphere inscribed in a square/cube we used a hit ratio to estimate the amount of space each object occupied (area/volume). This simple notion can than be applied to objects with higher dimensionality. By knowing the equations governing an objects boundaries and calculating its hit ratio we are able to estimate the "volume" of such an object, independent of what dimension it resides in.

1 Introduction

For this weeks lab we used the method of Monte-Carlo integration. This method allows us to calculate the volume of an object bounded within a given region, of known volume:

$$V_{unknown} = \frac{N_{hit}}{N_{tot}} V_{tot} \tag{1}$$

Using equation 1 allows for the approximation of a given volume by exploiting statistical probability. Figure 1 is a graphical representation of using the Monte-Carlo method of integration to estimate the volume of a sphere inscribed in our defined cube. The hit ratio is calculated by dividing the number of trials that landed within our selected boundary (the unit sphere) by the total number of trials. Since each trial is generted using a random number between -1 and 1 it has equal probability of hitting anywhere with our cube. By multiplying the hit ratio by the total volume of the cube, a value for the unknown volume of the unit sphere is obtained. As shown in figure 2 below the fractional area, when using this method, is minimized for a specific number of trials. This tells us that there is an ideal number of trials we should use to get the most accurate estimate possible.

2 Code

The code for this weeks can be seen online:
http://www2.hawaii.edu/∼cmutnik/lab5.html

3 Computational problem

Although we may not be able to envision an object residing in more that 3 spatial dimensions we can use the Monte-Carlo method of integration to calculate its "volume". First we must determine the equation governing its boundaries:

$$V_n(R) = \frac{\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2}+1)} R^n \tag{2}$$

This equation is used to calculate the true volume of an N-dimensional sphere [4]. In order to estimate such a volume we multiplied its hit ratio by the total area enclosing it.

4 Graphs

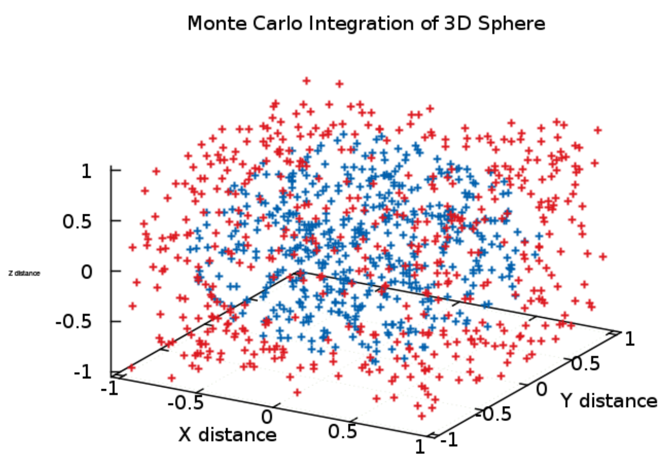

Figure 1: *Graphical representation of a Monte Carlo Integration for a 3D sphere*

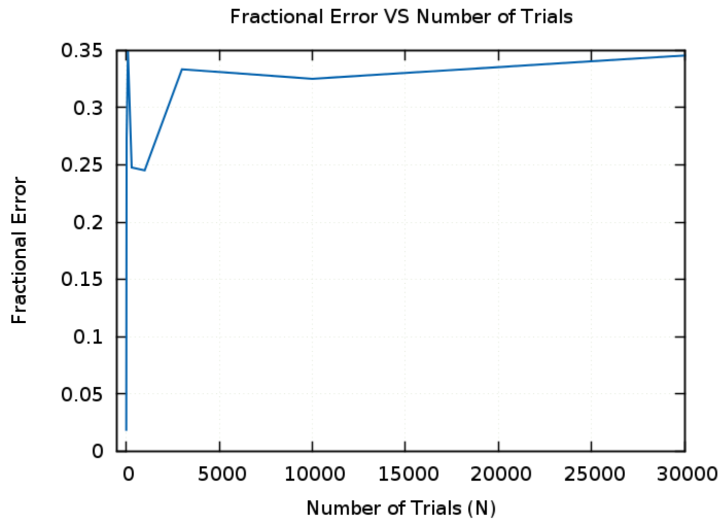

Figure 2: *Fractional error VS N*

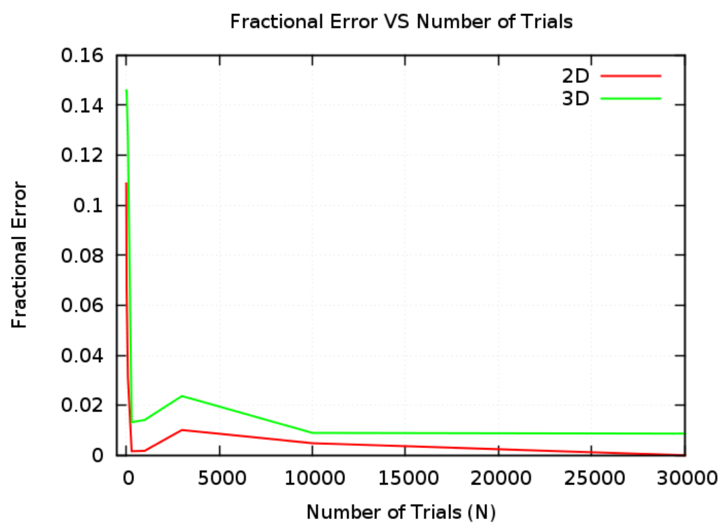

Figure 3: *Fractional error VS N for a 2D and 3D sphere*

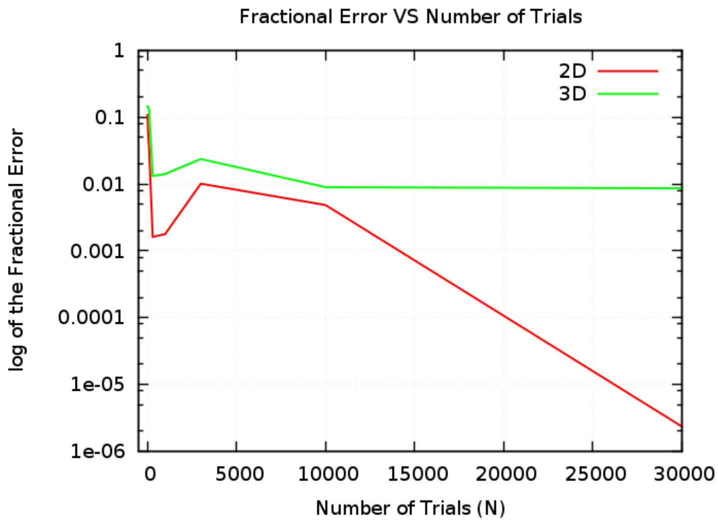

Figure 4: *Number of trials (N) VS log of fractional error for a 2D and 3D sphere*

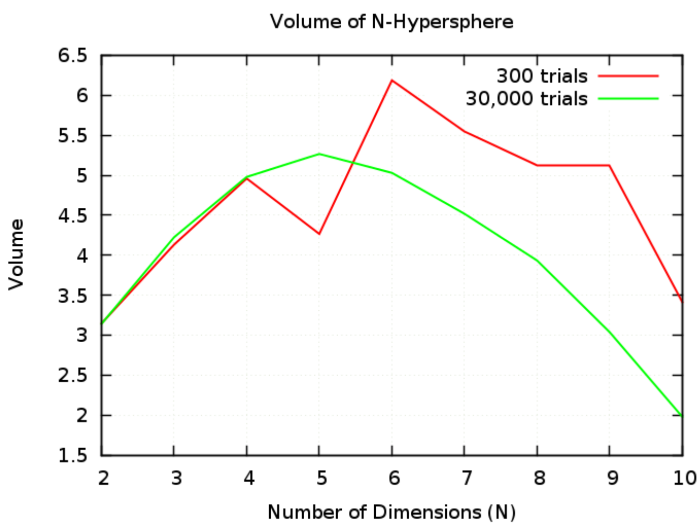

Figure 5: *Theoretical volume of a sphere as a function of dimensions for 300 and 30,000 trials*

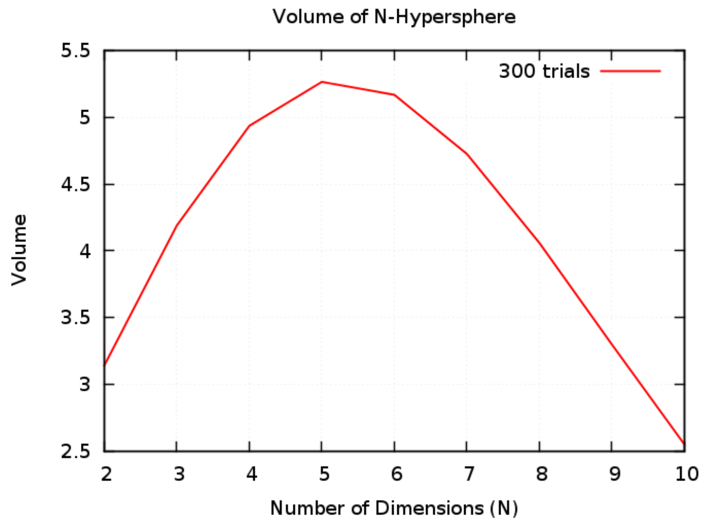

Figure 6: *Actual volume of sphere as a function of number of dimensions for 300 trials*

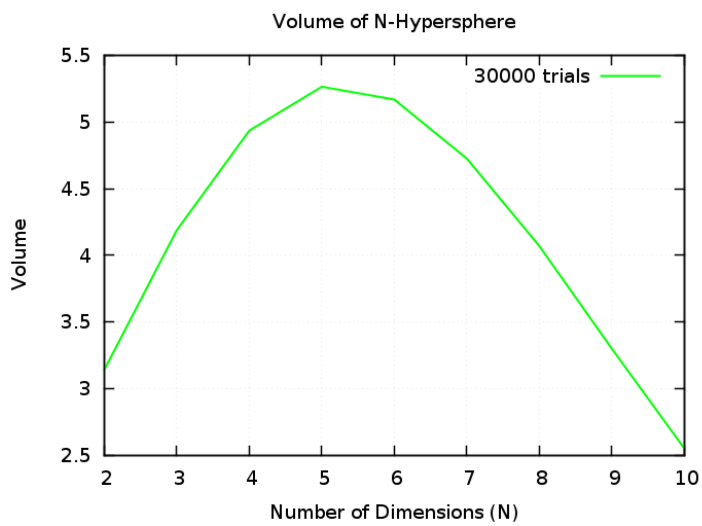

Figure 7: *Actual volume of sphere as a function of number of dimensions for 30,000 trials*

Dimensions	Vsphere	Vtrue	Analytical Value
2	3.1416	3.14159	3.14159
3	4.22507	4.18879	4.18879
4	4.97973	4.9348	4.93480
5	5.26613	5.26379	5.26379
6	5.0304	5.16771	5.167713
7	4.5184	4.72477	4.72477

This table was generated using 3,000 trials. Where Vsphere was calculated using the hit ratio, Vtrue was calculated using equation 2, and the analytical values were supplied[3].

5 Analysis

Figure 5 shows the volume as a funtion of dimensions for an n-dimensional sphere, using both 300 and 30,000 trials. 30,000 was selected since one expects the hit ratio to converge as the number of trials increases. 300 was selected since it was where the local minimum occured in the fractional error graph (figure 3).

In comparing our calculated values to the analytical values[3] two trends become apparent. The value of Vtrue is consistent with the analytic values, since equation 2 is a slight modification of the one used in generating them. As the number of dimensions increases one would expect the volume to increase as well. This is only the case up to five dimensions. The volume decreases after five dimensions. Since we can not take measurements in higher than three spatial dimensions we assume the volume of the hypersphere will increase as the volume of the hypercube does. Such an assumption is predicated on the trend observed to occur when the transition between one and three dimensions takes place. This assumption is either false or our equation governing such objects breaks down; its level of accuracy declines as more dimensions are added.

6 Conclusion

The Monte-Carlo method of integration is very useful in calculating the "volume" of an object when its boundaries are well defined. Although this method begins to break down for larger numbers of dimensions it is very useful in our physical world.

References

[1] R. H. Landau and M. J. Paez, "Computational Physics, Problem Solving with Computers," (Wiley: New York) 1997.

[2] P. Gorham, (2014).

[3] Enevoldsen, Keith. "N-Dimensions." Think Zone, n.d. Web. 12 Feb. 2015.

[4] Su, Francis. "Volume of a Ball in N Dimensions." *Mudd Math Fun Facts.* Harvey Mudd College Math Department, 1990. Web. 12 Feb. 2015.