Projectile Motion:
No-Dong Missile

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Abstract

Missiles launch from a surface using a propellent to accelerate. The duration of time during which a propellent is ignited, a burn time, determines the distance any such missile is able to travel. A single stage missile has one burn time. By accurately modeling the burn time the motion of such a rocket can be determined.

1 Introduction

From Newton's Second Law we know that the net force on an object is the rate of change of its momentum with respect to time. After little manipulation we can rewrite this force law to determine a characteristic equation for acceleration. Once this is done we are able to use computational methods, such as Runge-Kutta, to achieve numerical approximations that model system solutions.

$$\frac{d\vec{v}}{dt} = \frac{1}{m(t)}\left[\vec{F}_g(\vec{r}(t)) + \vec{F}_b(\vec{v}(t)) + \vec{F}_p(m(t))\right] \tag{1}$$

where m(t) is the time dependent mass, $\vec{F}_g$ is the force due to gravity, $\vec{F}_d$ is the force due to drag, and $\vec{F}_p$ is the force given to the rocket as a result of its thrust. The forces summed here are dependent on $\vec{r}(t)$, $\vec{v}(t)$, and m(t). Here $\vec{r}(t)$ is the 3-vector pertaining to the projectiles position, relative to the center of the Earth. The objects velocity is denoted by $\vec{v}(t)$.

$$\vec{F}(\vec{r}) = -\tfrac{GMm}{|\vec{r}|^3}\,\vec{r}$$

where G is the gravitational constant, M is the mass of the Earth, and m is the mass of the modeled projectile.

$$\vec{F}_b = -b\,|\vec{v}_{app}|\,\vec{v}_{app}$$

where $\vec{v}_{app}$ is the apparent velocity of the rocket. It takes into account the velocity due to the rockets motion as well as any needed modification imposed by the wind. Here b is Bernoulli's coefficient:

$$b = C_d A\rho/2 \tag{2}$$

where C_d is the drag coefficient, A is the cross sectional area, and the air density ρ is dependent on the magnitude of the radius vector (as measured from the center of the Earth).

In modeling the motion of a rocket it is not sufficient to use mass, one must use time dependent mass. As a rocket burns fuel its mass decreases. It is evident that once fuel is burned it no longer contributes to the mass in need of propulsion; any subsequent burning will be inherently more efficient. In order to properly model such behavior it is necessary to use a function for the mass of a missile rather than a constant value. The mass decreases in a linear fashion:

$$m(t) = m_0 - m_{fuel}\frac{t-t_0}{t_b},\ t < t_b$$

$$m(t) = m_0 - m_{fuel},\ t \geq t_b$$

where m_0 is the inital mass of the rocket (including the mass of fuel), m_{fuel} is the mass of fuel that has already been burned, t_0 is the inital time, and t_b is the burn time.

2 Computational problem

In order to model the projectile motion of a missile, simplifications needed to be made. First we assume the projectile to be a cylinder with a given diameter. This allows for a more easily calculable cross sectional area used in equation 2. Linear interpolation of atmospheric data allowed for the modeling of changing air density. This is crutial because the missile modeled here, the No-Dong missile, is shown to achieve altitudes that are in the order of hundreds of meters. At such heights it is bad practice to assume the air density constant.

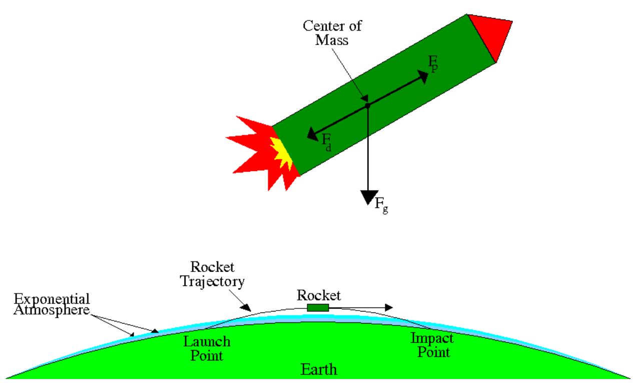

Figure 1: *Force diagram of rocket*

Figure 1 is a crude depiction of the projectile in this experiment. From the force diagram above one can see the direction of each force vector. The drag force opposes the motion of the rocket while the force from thrust is in the same direction as the rockets motion. As expected, the force from gravity points towards the center of Earth.

3 Results

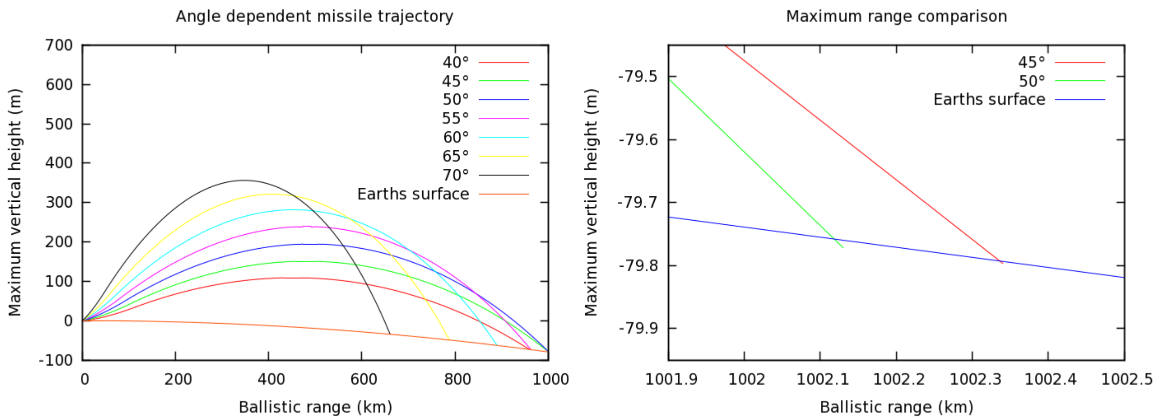

Figure 2: *Missile trajectory with launch angle dependency*

Depicted, on the left, in figure two are the trajectories of the missile as launched with varying inital angle. The inital angle values denoted in the figures key are taken from the horizontal. The graph on the right is a comparison of angles that achieve maximum range.

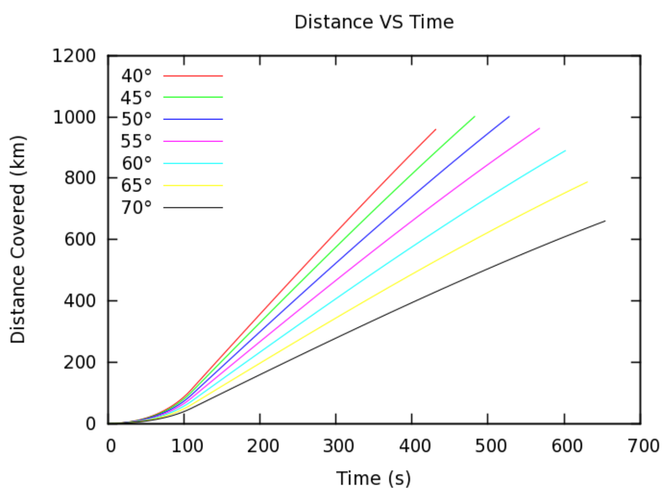

Figure 3: *Distance traveled over the ground as a function of time*

Figure 3 also shows that a missile fired at an angle of 45 degrees will travel downrange the most.

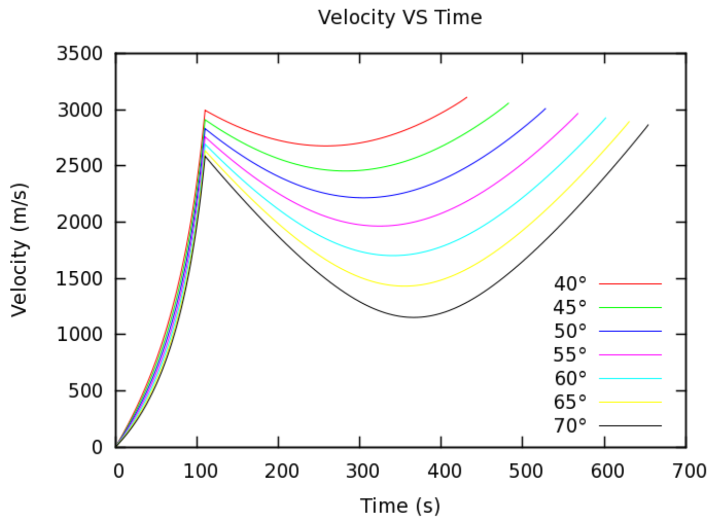

Figure 4: *Magnitude of the rockets velocity as a function of time*

4 Analysis

In order to achieve a maximum range this projectile must be fired at an angle of 45 degree from the launch surface. Figure 3 shows the distance traveled over the ground by the missile, as fired from various angles above the horizontal. Outside of the depicted angular range (40-70 degrees) the program breaks down. Due to gravitational dependencies such a simplified program is unable to accurately depict the missile lauched with and inital angle outside of this range. Using the inital launch parameters on the No-Dong missile allowed for the comparison of our results to published data. The modeled No-Dong missile achieved a maximum range of just over 1000 km. The maximum published range of the No-Dong missile, 700 - 1000 km, agrees with our results [3]. The trajectory parameters that give this maximum range are:

thrust: p = 26,051 kg

launch mass: m_0 = 16,000 kg

initial fuel mass: m_{fuel} = 12,912 kg

burn time: t_b = 110 s

diameter: 1.32 m

drag coefficient: C_d = 0.25

Figure 4 models the magnitude of the projectile's velocity as a function of time. At 110 seconds the burn ends, this is depicted by the local maximum in figure 4. From this point on the rocket enters free-fall. Eventually the acceleration due to gravity causes the rocket to change direction and begin its descent. This transition occurs at the local minimum in figure 4. The overall magnitude of the velocity begins to increase after this point, until the rocket reaches the impact point.

5 Conclusion

It is evident that a 45 degree angle achieves maximum downrange distance. Although simplifications were made, the program written to model a projectiles behavior do exceedingly well for the necessary range of angles. You can not ignore the effects that occur when the missile passes through the jet stream. The wind velocity in the jet stream causes a change in air density that is taken into account by interpolation atmospheric data. Any cross wind caused by the jet stream alters the path and velocity of the rocket. If one ignores the curvature of the Earth inaccurate results are achieved. Not accounting for the curvature of the Earth alters the landing point of the rocket. This causes a missile launched at a different inital angle to achieve the largest maximum range.

References

[1] R. H. Landau and M. J. Paez, "Computational Physics, Problem Solving with Computers," (Wiley: New York) 1997.

[2] Gorham, Peter. "Physics 305 Differential Equations." P305lab7. Phys.hawaii.edu, 6 March 2015. Web. 4 Apr. 2015.

[3] Vick, Charles P. "No-Dong." No-Dong 1 - North Korea. N.p., 17 Feb. 2015. Web. 06 Apr. 2015.

Acknowledgements

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